Numerical model for the determination of hysteresis loss through the drag force method

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***Abstract*— To determine the hysteresis loss in a sample, usually the enclosed area of the $B-H$ loop is evaluated. In the drag force method, based on a proper energy balance, the hysteresis loss is obtained by analyzing the drag force profile when slowly moving the sample forward and backward through the strong field of permanent magnets. A numerical time-stepping model is presented that calculates the drag force profile. At every time step, the sample is slightly moved. The model is based on 2D-FE computations including magnetic hysteretic material behaviour using the Preisach model. In order to improve the numerical stability, we reformulated the Maxwell equations in such a way that the material behaviour is described through differential permeabilities. Consequently, the basic unknown for the FEM becomes the time derivative of the vector potential. The drag force is obtained by using the Maxwell stress tensor. Simulations were carried out in which a sample was moved back and forth through the field of one or two permanent magnets. The numerical model is validated by measurements and by comparing the "drag force" hysteresis losses with the hysteresis losses computed conventionally by the integral of $H.\mathrm{d}B$.**

I. INTRODUCTION

In the drag force method, the hysteresis loss is obtained by measuring the drag force that arises from the forward and backward movement of a sample relative to the strong field of permanent magnets [1]. The distance between magnet and sample is kept constant in time. In the calculation, the magnet is moved to the left over the fixed sample – see Fig. 1a. To avoid eddy currents, the movement should be sufficiently slow.

The drag force profile is calculated by a numerical time-stepping model that uses a finite element model (FEM) combined with a hysteresis model. We first focus on these two models and then explain the whole calculation scheme.

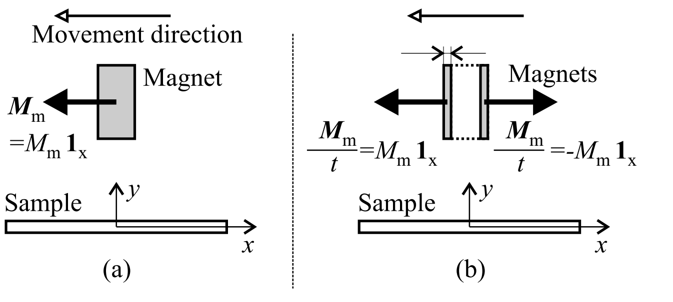

Fig. 1. (a) Geometry of the drag force method (not on scale) in case of one magnet that moves over the sample to the left. (b) Implementation of the method in the model using differential permeability. The moving magnet is replaced by two thin magnet slices with the movement step Δ as thickness.

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II. NUMERICAL MODEL

A. FEM with differential permeability

As there are no current sources in the problem, the classical FEM with vector potential $\mathbf{A}$ – where $\mathbf{B}=\nabla\times\mathbf{A}$ – solves $\nabla\times\left(\frac{1}{\mu}\nabla\times\mathbf{A}\right)=0$ or $\nabla\times(\nabla\times\mathbf{A})=\mu_0(\nabla\times\mathbf{M})$. In a problem with hysteresis, it is not possible to solve the vector potential FE problem using the classical permeability, because this permeability may become negative, zero or infinite. To overcome this, the problem is rewritten in terms of the time derivative of $\mathbf{A}$. In the magnet, the equation to solve is:

$$\nabla\times\left(\nabla\times\frac{\partial\mathbf{A}}{\partial t}\right)=\mu_0\left(\nabla\times\frac{\partial\mathbf{M}_\mathrm{m}}{\partial t}\right)\tag{1}$$

with $\mathbf{M}_\mathrm{m}$ the magnetization of the magnet (see Fig. 1a). For the sample, we introduce the differential permeability $\bar{\bar{\mu}}_\mathrm{d}$ that relates the time derivatives of the flux density and the magnetic field vectors $\mathbf{B}$ and $\mathbf{H}$: $\frac{\partial\mathbf{B}}{\partial t}=\bar{\bar{\mu}}_\mathrm{d}\frac{\partial\mathbf{H}}{\partial t}$. In the sample we have:

$$\nabla\times\left(\frac{1}{\bar{\bar{\mu}}_\mathrm{d}}\nabla\times\frac{\partial\mathbf{A}}{\partial t}\right)=\mathbf{0}\tag{2}$$

The flux density at time instant $t_k+\Delta t$ is obtained by adding $\frac{\partial\mathbf{B}}{\partial t}\cdot\Delta t$ to the previous $\mathbf{B}$-value at t_k. The differential permeability is function of the position in the sample. Its distribution is determined by the hysteresis model (see next paragraph) based on the magnetic field history. As the distribution of $\bar{\bar{\mu}}_\mathrm{d}$ only depends on the previous field values and as eddy currents are not taken into account, the FEM is static.

B. Preisach hysteresis model in FEM

The differential permeability $\bar{\bar{\mu}}_\mathrm{d}$ is described by the classical scalar Preisach model [3]. For the studied material, an Everett map is generated by measuring a high number of hysteresis loops in a single sheet tester. For a sequence of scalar H values, the corresponding differential permeabilities are calculated by the derivatives of the Everett map [4]. All relevant minima H_min and maxima H_max are stored in memory. This hysteresis model is combined with the FEM. To handle the 2D-behaviour of the magnetic field vector $\mathbf{H}$, the following approximation is considered. As in the drag force method with configuration of Fig. 1a, the field is mainly along the x-axis, the scalar hysteresis model is applied only to the x-component of $\mathbf{H}$. For H_y, a nonlinear characteristic without hysteresis is used. The non-diagonal elements of tensor $\bar{\bar{\mu}}_\mathrm{d}$ are 0.

C. Time-stepping procedure

After the initialization procedure in Fig. 2, the magnet is moved backward and forward. During the corresponding time stepping procedure, $t_k=t_{k-1}+\Delta t$, $k=1,2,3,\ldots$, equation

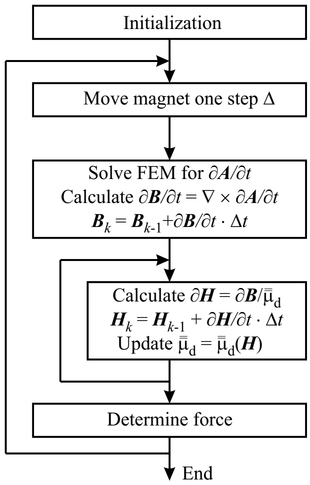

Fig. 2. Scheme of the numerical model

(1) must be solved at each time point k. At time step k, the magnet is slightly moved over a distance Δ. Here, the movement of the magnet results in two subregions with width Δ where $\frac{\partial \mathbf{M}_\mathrm{m}}{\partial t} \neq 0$. These two subregions are defined by the position of the magnet at time point $k-1$ and time point k – see Fig. 1b. As dynamic effects are not taken into account, we may take for simplicity $\Delta t = 1$. Hence, for the 2 subregions in Fig. 1b, $\frac{\partial \mathbf{M}_\mathrm{m}}{\partial t}$ becomes $M_m\ \mathbf{1}_x$ and $-M_m\ \mathbf{1}_x$ respectively.

The distribution $\bar{\bar{\mu}}_\mathrm{d}$ in (2) in the sample at time t_k is found by the hysteresis model using H_{k-1} and previous extrema.

The FE problem is solved resulting in the solution $\frac{\partial \mathbf{A}}{\partial t}$ and $\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \frac{\partial \mathbf{A}}{\partial t}$. The flux density $\mathbf{B}_k = \mathbf{B}_{k-1} + \frac{\partial \mathbf{B}}{\partial t} \cdot \Delta t$ is updated. Now, $\mathbf{H}_k = \mathbf{H}_{k-1} + \frac{1}{\bar{\bar{\mu}}_\mathrm{d}} \frac{\partial \mathbf{B}}{\partial t} \cdot \Delta t$ and $\bar{\bar{\mu}}_\mathrm{d}(\mathbf{H}_k)$ are updated iteratively. Finally, the drag force at time k is obtained by integrating the Maxwell stress tensor applied to $\mathbf{H}_k$ along a path in the air surrounding the sample at a small distance.

We define a trajectory of motion during which the magnet is moved backward and forward with constant speed and for which the start and the end position of the magnet are the same. The hysteresis loss can be obtained by the energy balance:

$$\oint_\mathrm{traject} F_x \cdot \mathrm{d}x = \int_V \left(\oint_\mathrm{loop} \mathbf{H} \cdot \mathrm{d}\mathbf{B} \right) \mathrm{d}V \tag{3}$$

The work done by the drag force F_x along the trajectory of motion is equal to the conventional losses integrated over the whole volume of the sample. Consequently, (3) allows to estimate the hysteresis losses in the sample by measuring the drag force F_x.

III. Experimental validation and conclusion

Simulations were carried out for the geometry of the experimental setup [1] in which one NdFeB permanent magnet was moved back and forth over a sample of 1.6 mm thickness. The magnet is positioned as in Fig. 1a. Fig. 3a shows the drag force profile during the movement, and the arrows indicate the movement direction of the magnet. Fig. 3b shows the same profile as a function of the magnet position. The corresponding $B-H$ behaviour in the center of the sample is shown in Fig. 3c. Initially, the magnet is at the right of the sample in position (1). The dotted line in Fig. 3a and b shows that F_x in N per meter length in z-direction becomes high (264 N/m) as the magnet is moved to the left, comes closer to the 15 cm long sample and attracts it. Then, the force profile becomes more or less flat up to position (2). Fig. 3c shows that the considered point (the center of the sample) is magnetized until saturation and back to a negative H in (2). Now, the movement direction is changed and we follow the solid line along (3) to position (4). The force is higher due to hysteresis. Finally, the movement direction is changed again. The sample is brought back to position (5)=(2), see Fig. 3b. The area enclosed by the solid line in Fig. 3b indicates the hysteresis loss. The method is validated numerically by checking the equality of both sides of eq. (3). The correspondence was good.

Similar to the experiments in [2], also simulations were done with two magnets, symmetrical to the $y=0$ plane. The calculated drag force profile is similar to the experimental one shown in [2], with the advantage that the huge normal force component F_y (547 N/m) appearing in the one-magnet setup is cancelled.

The full paper will describe a four-magnet configuration. The two extra magnets magnetize in opposite direction, so that the hysteresis loops range from $-\mathbf{B}_\mathrm{sat}$ to $+\mathbf{B}_\mathrm{sat}$, while the one and two magnet configurations achieve $+\mathbf{B}_\mathrm{sat}$ only.

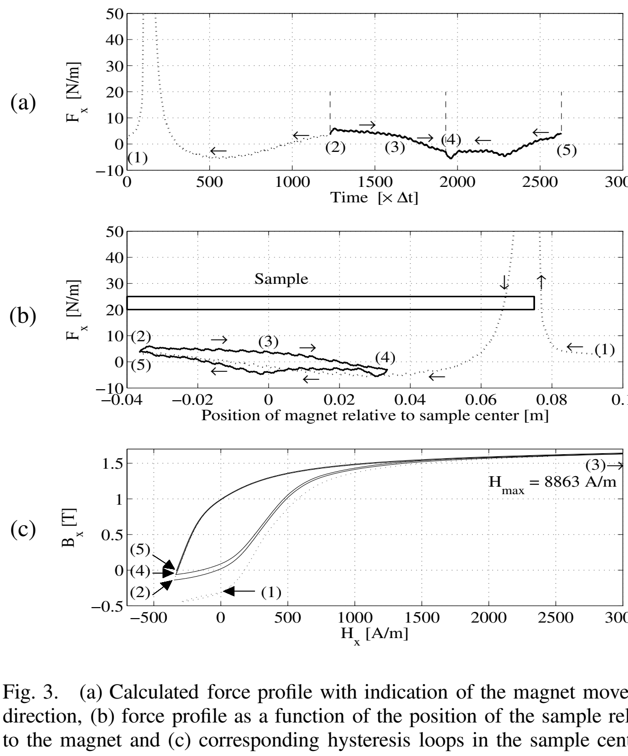

Fig. 3. (a) Calculated force profile with indication of the magnet movement direction, (b) force profile as a function of the position of the sample relative to the magnet and (c) corresponding hysteresis loops in the sample center.

References

[1] I. J. Garshelis, S. P. L. Tollens, R. J. Kari, L. Vandenbossche, and L. Dupré, "Drag force measurement: A means for determining hysteresis loss," *J. Appl. Phys.*, Vol. 99, Art. No. 08D910, Apr. 2006.

[2] I. Garshelis, S. Tollens, L. Vandenbossche, et al., "Determination of hysteresis loss by drag force measurements with cancelled normal field components," *Intermag Conf.*, 8–12 May 2006, San Diego, CA, p. 593.

[3] I. Mayergoyz, *Mathematical Models of Hysteresis and their Applications*, Academic Press, 2003.

[4] D. Everett, "A general approach to hysteresis - part 4, An alternative formulation of the domain model," *Transactions Faraday Society*, Vol. 51, pp. 1551–1557, 1955.